6.034
Problem Solving Strategies and Methods
Randall Davis (friend of Dr. K)

Outline
- Task: Integration
- Examine the method
- Examine the knowledge
- Understand how SAINT, a program from 1960 worked. (1960!)
- Why failures are wonderful
- What do you need to know to be good at something?

Major ideas
- Knowledge is power
  - What kind
  - How much
  - How represented
  - How used
  - What exactly do we need to know
- Collect good ideas
- The power of building models

The Task
\[ \int \frac{-5x^4}{(1-x^2)^{5/2}} \, dx \]
How Would You Approach It?
How Do We Do It?

- $\int \frac{1}{x} \, dx = ?$
- $\int x^n \, dx = ?$
- $\int \cos x \, dx = ?$
- and...

Architecture

```
Apply safe xforms  Check table  DONE?  ☺
Apply heur. transform  Select problem
```

\[
\int -5 \frac{x^4}{(1-x^2)^{5/2}} \, dx = -5 \int \frac{x^4}{(1-x^2)^{5/2}} \, dx = -5 \int \frac{\sin^4(y)}{\cos^4(y)} \, dy
\]

\[
\frac{\tan^4y \cdot dy}{1 \cdot \cot^4y \cdot dy} = \frac{32}{(1+z^2)(1-z^2)^2} \, dz
\]

\[
\frac{z^4}{1+z^2} \, dz - \int \frac{dz}{z^2(1+z^2)}
\]

\[
\left[ \frac{1+z^2}{1+z^2} + \frac{1}{1+z^2} \right] \, dz
\]

\[
-\frac{z^4}{3} + \frac{1}{1+z^2} \quad \text{try } w = \arctan z
\]

\[
\arcsin(x) - \tan(\arcsin(x)) + \frac{1}{3} \tan^3(\arcsin(x))
\]

\[
\int \frac{-5 x^4}{(1-x^2)^{5/2}} \, dx =
\]

\[
-5 \left( \arcsin(x) - \tan(\arcsin(x)) + \frac{1}{3} \tan^3(\arcsin(x)) \right)
\]
(Why) Is This Interesting?
- Notion of *problem reduction*
- Goal tree
- And-node, or-node

The Power of Naming Things
- (Folklore)
  - Ancient Egypt, Jewish tradition, Rumplestiltskin
- (Literature)
  - *The Nine Billion Names of God*, Clarke
  - *A Wizard of Earthsea*, Le Guin
  - ...
- Engineering
  - *Reify* a vague notion into a concrete concept
  - Call on it when and how you will.

And What Of It?
- Evaluating performance
  - 54 of 56
  - *Misstakes are wonderful*

Knowledge
- What kind
  - Transforms
  - Goal trees
- How represented
  - Tables
- How used
  - Xforms for problem reduction
  - Tables for primitive problem solution
Knowledge

- How much
  - 24 transforms
    - 12 safe
    - 12 heuristic

The Mindset Of SAINT

- Worked like the average engineer, i.e., lots of search and backtracking
- Conceived of in terms of search, worked because of that. The power comes from:
  - Problem decomposition
  - Methodical exploration of alternatives
  - Looking far, wide, and deep
  - Speedy tree construction, search, backtracking
- Success is just a matter of trying enough alternatives

An Important Lesson

- The power of building models
- Especially executable models

Some Stats & An Inconvenient Truth

- Statistics
  - Max depth of tree: 7
  - Average depth: ~3
  - Unused branches: ~1
- How many rules on average applicable to an expression?
  - 1
- In consequence of that truth: SIN
  - We ...mainly desired a powerful integration program which behaved closely to our conception of expert human integrators.