6.034

Problem Solving Strategies and Methods

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Outline

- Task: Integration
- Examine the method
- Examine the knowledge
- Understand how SAINT, a program from 1960 worked. (*1960!*)
- Why failures are *wonderful*
- What do you need to know to be good at something?

Major ideas

- ★Knowledge is power
  - What kind
  - How much
  - How represented
  - How used
  - What exactly do we need to know
- ★Collect good ideas
- ★The power of building models

The Task

$$\int \frac{-5\,x^4}{(1-x^2)^{5/2}}\,dx$$

How Would You Approach It?

How Do We Do It?

- $\int \frac{1}{x} dx = ?$
- $\int x^n \, dx = ?$
- $\int \cos x = \ ?$
- and…

$$\int \frac{-5\,x^4}{(1-x^2)^{5/2}} \mathrm{dx} \Rightarrow -5 \int \frac{x^4}{(1-x^2)^{5/2}} \mathrm{dx} \Rightarrow -5 \int \frac{sin^4(y)}{cos^4(y)} \mathrm{dy}$$

x=sin(y)

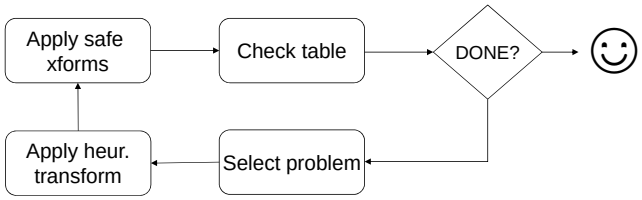
Z=sin(y)

$\int tan^4 y \ \mathrm{dy}$ → Heur. B → $\int \frac{z^4}{1+z^2} dz$ → Divide → $\int \left(-1+z^2+\frac{1}{1+z^2}\right) dz$ → $-z+\frac{z^3}{3}+\int \frac{dz}{1+z^2}$ → try w = arctan z → $\arcsin(x)-\tan(\arcsin(x))+\frac{1}{3}\tan^3(\arcsin(x))$

$\int 1/cot^4 y \ \mathrm{dy}$ → $-\int \frac{dz}{z^4(1+z^2)}$

$\int 32 \frac{z^4}{(1+z^2)(1-z^2)^4} dz$

Architecture



$$\int \frac{-5\,x^4}{(1-x^2)^{5/2}} \mathrm{dx} =$$

$$-5 * (\arcsin(x) - \tan(\arcsin(x)) + \tfrac{1}{3} tan^3(\arcsin(x)))$$

(Why) Is This Interesting?

- Notion of *problem reduction*
- Goal tree
- And-node, or-node

The Power of Naming Things

- (Folklore)
  - Ancient Egypt, Jewish tradition, Rumplestiltskin
- (Literature)
  - *The Nine Billion Names of God,* Clarke
  - *A Wizard of Earthsea,* Le Guin
  - …
- Engineering
  - *Reify* a vague notion into a concrete concept
  - Call on it when and how you will.

And What Of It?

- Evaluating performance
  - 54 of 56
  - ***Misstakes are wonderful***

Knowledge

- What kind
  - Transforms
  - Goal trees
- How represented
  - Tables
- How used
  - Xforms for problem reduction
  - Tables for primitive problem solution

Knowledge

- How much
  - 24 transforms
    - 12 safe
    - 12 heuristic

An Important Lesson

- The power of building models
- Especially *executable* models

The Mindset Of SAINT

- Worked like the average engineer, i.e., lots of search and backtracking
- Conceived of in terms of search, worked *because* of that. The power comes from:
  - Problem decomposition
  - Methodical exploration of alternatives
  - Looking far, wide, and deep
  - Speedy tree construction, search, backtracking
- Success is just a matter of trying enough alternatives

Some Stats & An Inconvenient Truth

- Statistics
  - Max depth of tree: 7
  - Average depth: ~3
  - Unused branches: ~1
- How many rules on average applicable to an expression?
  - *1*
- In consequence of that truth: SIN
  - *We …mainly desired a powerful integration program which behaved closely to our conception of expert human integrators.*