6.034
Problem Solving Strategies and Methods

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Outline

- Task: Integration
- Examine the method
- Examine the knowledge
- Understand how SAINT, a program from 1960 worked. (*1960!*)
- Why failures are *wonderful*
- What do you need to know to be good at something?

Major ideas

- Knowledge is power
  - What kind
  - How much
  - How represented
  - How used
  - What exactly do we need to know
- Collect good ideas
- The power of building models

The Task

$$\int \frac{-5\,x^4}{(1-x^2)^{5/2}}\,\mathrm{dx}$$

How Would You Approach It?

How Do We Do It?

- $\int \frac{1}{x}\, dx = ?$
- $\int x^n\, dx = ?$
- $\int \cos x = \ ?$
- and…

$$\int \frac{-5\, x^4}{(1-x^2)^{5/2}} dx \Rightarrow -5 \int \frac{x^4}{(1-x^2)^{5/2}} dx \Rightarrow -5 \int \frac{sin^4(y)}{cos^4(y)} dy$$

x=sin(y)

$$\int tan^4 y\, dy \qquad \int 1/cot^4 y\, dy$$

Z=sin(y)

$$\int 32 \frac{z^4}{(1+z^2)(1-z^2)^4} dz$$

Heur. B

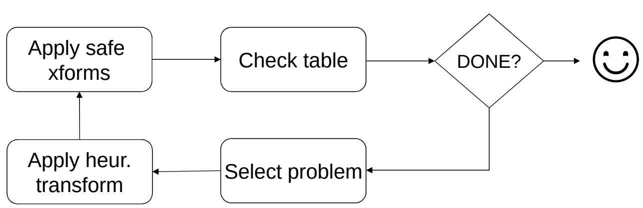
$$\int \frac{z^4}{1+z^2} dz \qquad -\int \frac{dz}{z^4(1+z^2)}$$

Divide

$$\int \left(-1+z^2+\frac{1}{1+z^2}\right) dz$$

$$-z+\frac{z^3}{3}+\int \frac{dz}{1+z^2} \xrightarrow{\text{try } w = \arctan z} \arcsin(x)-\tan(\arcsin(x))+\frac{1}{3}\tan^3(\arcsin(x))$$

Architecture



$$\int \frac{-5\, x^4}{(1-x^2)^{5/2}}\, dx =$$

$$-5 * (\arcsin(x) - \tan(\arcsin(x)) + \tfrac{1}{3} tan^3(\arcsin(x)))$$

(Why) Is This Interesting?

- Notion of *problem reduction*
- Goal tree
- And-node, or-node

The Power of Naming Things

- (Folklore)
  - Ancient Egypt, Jewish tradition, Rumplestiltskin
- (Literature)
  - *The Nine Billion Names of God,* Clarke
  - *A Wizard of Earthsea,* Le Guin
  - …
- Engineering
  - *Reify* a vague notion into a concrete concept
  - Call on it when and how you will.

And What Of It?

- Evaluating performance
  - 54 of 56
  - ***Misstakes are wonderful***

Knowledge

- What kind
  - Transforms
  - Goal trees
- How represented
  - Tables
- How used
  - Xforms for problem reduction
  - Tables for primitive problem solution

Knowledge

- How much
  - 24 transforms
    - 12 safe
    - 12 heuristic

An Important Lesson

- The power of building models
- Especially *executable* models

The Mindset Of SAINT

- Worked like the average engineer, i.e., lots of search and backtracking
- Conceived of in terms of search, worked *because* of that. The power comes from:
  - Problem decomposition
  - Methodical exploration of alternatives
  - Looking far, wide, and deep
  - Speedy tree construction, search, backtracking
- Success is just a matter of trying enough alternatives

Some Stats & An Inconvenient Truth

- Statistics
  - Max depth of tree: 7
  - Average depth: ~3
  - Unused branches: ~1
- How many rules on average applicable to an expression?
  - *1*
- In consequence of that truth: SIN
  - *We …mainly desired a powerful integration program which behaved closely to our conception of <u>expert human integrators</u>.*

Another Inconvenient Truth

- Technical papers are often badly written

Saint's Average Performance

	Subgoals	Unused Subgoals	Level	Heuristic Level
32 Author problem	6.4	2.0	3.5	1.0
52 MIT Problems	4.7	0.8	2.9	.8
84 Problems	5.3	1.25	3.0	.9

- Be **bold**

From: James Slagle <jrslagle@houston.rr.com>
To: Randy Davis <davis@ai.mit.edu>
Subject: Re: blast from the past

davis@ai.mit.edu wrote:
> Is "heuristic level" a count of the max *number* of heuristics used
> in a successful branch (as your previous example suggests), or
> is it the max *depth* at which a heuristic was used on a successful branch?
>
> thanks
> R.

It is the former.
More precisely, over all paths (branches) in the solution tree found by SAINT, heuristic level is the max of the number of heuristic transformations used in the path.
More questions?
Jim Slagle

Is SAINT intelligent?