Outline

- Task: Integration
- Examine the method
- Examine the knowledge
- Understand how SAINT, a program from 1960 worked. (1960!)
- Why failures are wonderful
- What do you need to know to be good at something?

Major ideas

- Knowledge is power
  - What kind
  - How much
  - How represented
  - How used
  - What exactly do we need to know
- Collect good ideas
- The power of building models

The Task

\[ \int \frac{-5x^4}{(1-x^2)^{5/2}} \, dx \]

How Would You Approach It?
How Do We Do It?

- \( \int \frac{1}{x} \, dx = ? \)
- \( \int x^n \, dx = ? \)
- \( \int \cos x = ? \)
- and…

Architecture

Apply safe xforms → Check table → DONE? 
Apply heur. transform → Select problem

\[
\int -5 x^4 \, dx = -5 \int \frac{x^4}{(1-x^2)^{5/2}} \, dx = -5 \int \frac{\sin^4(y)}{\cos^4(y)} \, dy
\]

Heur. B

\[
\int \tan^4 y \, dy \quad \int 1/\cot^4 y \, dy
\]

Divide

\[
\int \frac{z^4}{1+z^2} \, dz = -\int \frac{dz}{z^2(1+z^2)}
\]

\[
\int \left( -1+z^2 + \frac{1}{1+z^2} \right) \, dz
\]

try \( w = \arctan z \)

\[
-\frac{z^5}{3} + \frac{1}{\ln(z^2)} \quad \arcsin(x) - \tan(\arcsin(x)) + \frac{1}{3} \tan^3(\arcsin(x))
\]

\[
\int -5 x^4 \, dx = -5 \left( \arcsin(x) - \tan(\arcsin(x)) + \frac{1}{3} \tan^3(\arcsin(x)) \right)
\]
(Why) Is This Interesting?
- Notion of problem reduction
- Goal tree
- And-node, or-node

The Power of Naming Things
- (Folklore)
  - Ancient Egypt, Jewish tradition, Rumplestiltskin
- (Literature)
  - *The Nine Billion Names of God*, Clarke
  - *A Wizard of Earthsea*, Le Guin
  - ...
- Engineering
  - *Reify* a vague notion into a concrete concept
  - Call on it when and how you will.

And What Of It?
- Evaluating performance
  - 54 of 56
  - *Misstakes are wonderful*

Knowledge
- What kind
  - Transforms
  - Goal trees
- How represented
  - Tables
- How used
  - Xforms for problem reduction
  - Tables for primitive problem solution
Knowledge

- How much
  - 24 transforms
    - 12 safe
    - 12 heuristic

The Mindset Of SAINT

- Worked like the average engineer, i.e., lots of search and backtracking
- Conceived of in terms of search, worked because of that. The power comes from:
  - Problem decomposition
  - Methodical exploration of alternatives
  - Looking far, wide, and deep
  - Speedy tree construction, search, backtracking
- Success is just a matter of trying enough alternatives

An Important Lesson

- The power of building models
- Especially executable models

Some Stats & An Inconvenient Truth

- Statistics
  - Max depth of tree: 7
  - Average depth: ~3
  - Unused branches: ~1
- How many rules on average applicable to an expression?
  - 1
- In consequence of that truth: SIN
  - We ...mainly desired a powerful integration program which behaved closely to our conception of expert human integrators.
Another Inconvenient Truth

- Technical papers are often badly written

<table>
<thead>
<tr>
<th>Subgoals</th>
<th>Unused Subgoals</th>
<th>Level</th>
<th>Heuristic Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 Author problem</td>
<td>6.4</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td>52 MIT Problems</td>
<td>4.7</td>
<td>0.8</td>
<td>2.9</td>
</tr>
<tr>
<td>84 Problems</td>
<td>5.3</td>
<td>1.25</td>
<td>3.0</td>
</tr>
</tbody>
</table>

- Be bold

Is SAINT intelligent?

From: James Slagle <jrslagle@houston.rr.com>
To: Randy Davis <davis@ai.mit.edu>
Subject: Re: blast from the past

davis@ai.mit.edu wrote:
- Is "heuristic level" a count of the max *number* of heuristics used
- in a successful branch (as your previous example suggests), or
- is it the max *depth* at which a heuristic was used on a successful branch?

> > thanks

> R.

It is the former.
More precisely, over all paths (branches) in the solution tree found by
SAINT, heuristic level is the max of the number of heuristic transformations
used in the path.
More questions?
Jim Slagle