6.034

Constraints and Resource Allocation

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Gold Star Ideas

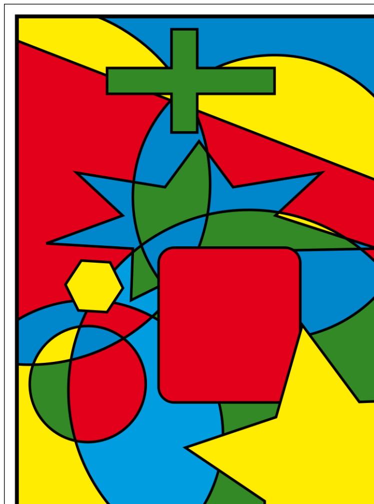
- Martial Arts Principle
  - Use enemy's strength against him
- Any-time Algorithms
-

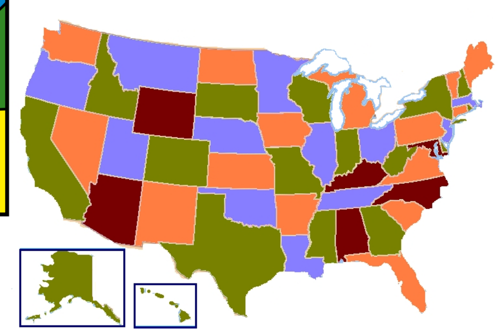
Four Color Theorem

- "given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color"
  - ignoring lakes, discontinuous states
- proved in 1976 by Kenneth Appel and Wolfgang Haken
  - using computer programs to show 1,936 cases and a 400+ page proof
- (five colors adequate proven in 1800s)

https://en.wikipedia.org/wiki/Four_color_theorem

Four-Colorings





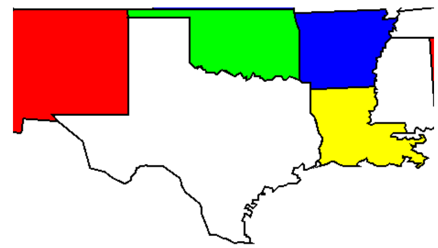
https://en.wikipedia.org/wiki/Four_color_theorem

How?

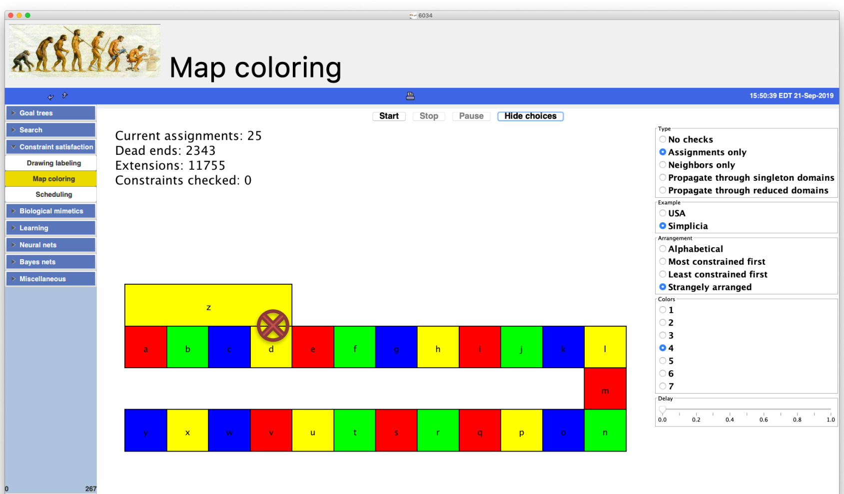
- Pick some order of states
- Choose four colors in rotation
  - {Red, Blue, Green, Yellow}
- Depth-first search
  - Main question: When can you tell that a path (partial coloring) is a “loser”?

Problem

- We may create a no-good situation early in the search, but not recognize it until very late in the game
- Consider coloring TX, NM, OK, AK, LA



Simplicia



Vocabulary

- Variable V: something that can have an assignment
- Value x: something that can be assigned
- Domain D: a bag of values
- Constraint C: a condition that must be satisfied among variable values

Systematic Idea for Map Coloring: *Domain Reduction Algorithm*

- For each depth first search assignment
  - For each variable V_i ***considered*** (we have choices here)
    - For each value x_i in D_i (domain of V_i)
      - For each constraint C between V_i and other variables V_j (we use binary constraints)
        - If $\nexists x_j \in D_j$ such that $C(x_i, x_j)$ is satisfied
        - Then remove x_i from D_i

What Do We "Consider"? (case of strangely arranged states)

Consider	dead ends	extensions	constraints checked
Nothing (wrong answer)	0	48	0
Assignment	$4^{48}/2$	≈∞	0
Neighbors only	406	2113	4667
Propagate through singleton domains	0	75	585
Propagate through reduced domains	0	75	2095
Everything			

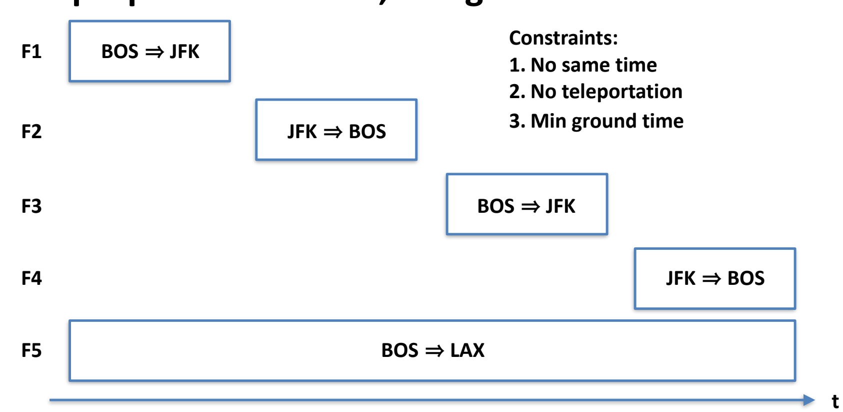
What Do We "Consider"?

ordering of states: strange, alphabetic, most, least constrained

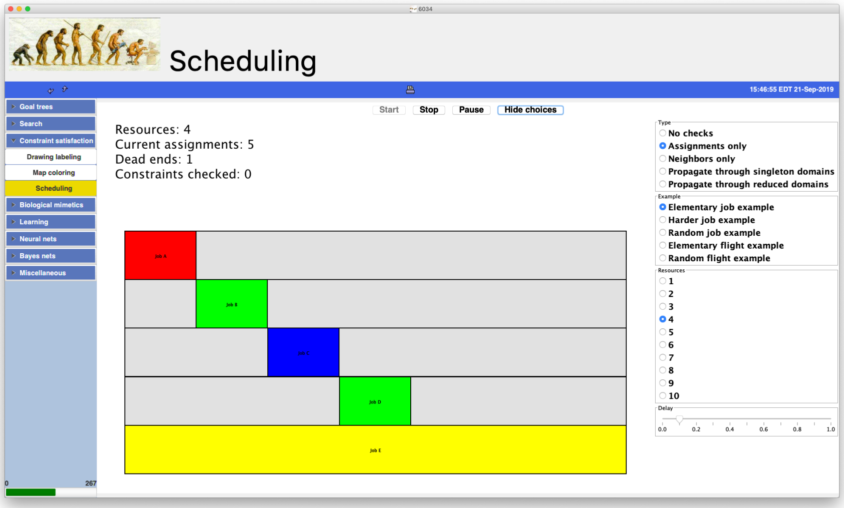
Consider	dead ends	extensions	constraints checked	
Assignment	≈∞ 1827 3 ≈∞	≈∞ 9217 101 ≈∞	0 0 0 0	s a m l
Neighbors only	406 0 0 1371	2113 82 86 6945	4667 244 224 10302	s a m l
Propagate through singleton domains	0 0 0 0	75 82 86 82	585 492 299 492	s a m l
Propagate through reduced domains	0 0 0 0	75 82 86 82	2095 2074 1725 2074	s a m l

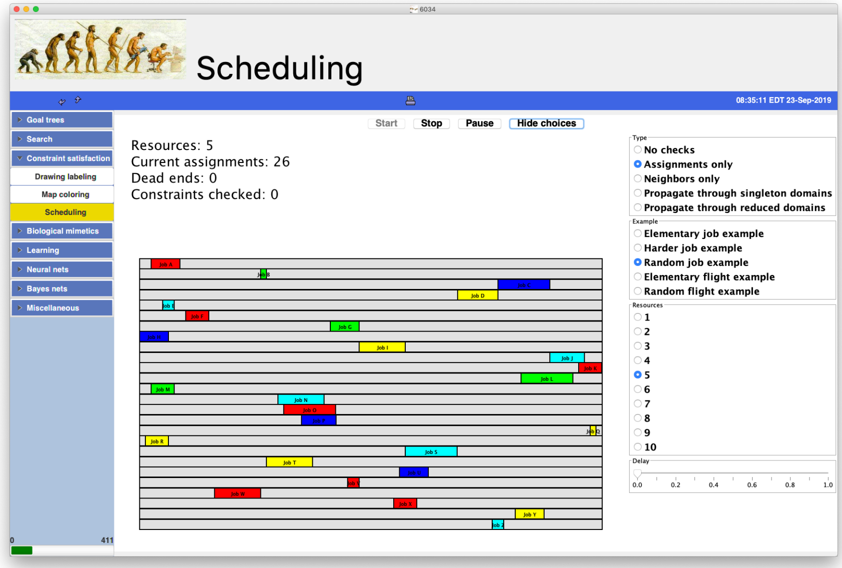
Resource Allocation

- Consider an airline with the following proposed schedule, using 4 aircraft:



JetGreen Airlines





Many Constraint Satisfaction Problems

SEND
MORE
MONEY

A store sells two types of toys, A and B. The store owner pays $8 and $14 for each one unit of toy A and B respectively. One unit of toys A yields a profit of $2 while a unit of toys B yields a profit of $3. The store owner estimates that no more than 2000 toys will be sold every month and he does not plan to invest more than $20,000 in inventory of these toys. How many units of each type of toys should be stocked in order to maximize his monthly total profit profit?